

IG LAB: gravity and inverse square law Name: _____

Gravity is a great real life application of the inverse (indirect) square law, or, 'y is inversely proportional to the square of x'. The gravity field of a body (satellite, planet, baseball) is inversely proportional to the distance from the centre of the body squared. As an equation this is:

$$F = \frac{Gm_1m_2}{r^2} \rightarrow g \propto \frac{1}{r^2} \Rightarrow g = \frac{k}{r^2}$$

g = Little-g or gravitational acceleration field of mass (m s^{-2})

r = distance from centre of the body (meters (m))

F = force (Newtons (N))

G = Big-G or Gravitational constant ($\text{m}^3/\text{kg}\cdot\text{s}^2$)

Task 1: the k constant

For each of the planets listed below, use the figures for surface gravity and radius to find the k constant.

Planet	Mercury	Venus	Earth	Mars	Jupiter
Surface Gravity (ms^{-2})	3.7	8.87	9.81	3.69	24.79
Radius (1000 m)	2,439	6,051	6,378	3,396	71,492

Task 2: the mass

In each of your equations the constant k is the mass (m) of each planet multiplied by the gravitational constant (G). The form of the equation is:

$$k = G * m \text{ (units ?)} \quad \text{hence} \quad g = \frac{Gm}{r^2} \left(\frac{m}{s^2} \right)$$

The gravitational constant is a very small number when expressed in MKS units

$$G = 6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

We can use this to find the mass of each planet. Divide each of your constant k values by G to find the mass (in kg) of each planet.

Questions (show your work please)

- (5 pt) For the five planets, show your work to calculate the k and mass values for the planets.
- (2 pt) The Earth orbits the Sun at an average distance of 150 billion metres (from their centers) and the Sun's mass is 1.99×10^{30} kg. What is the gravitational acceleration of the Earth towards the Sun due to the Sun's gravitational field?

3. (2 pt) What is the gravitational acceleration of the Sun towards the Earth as a result of the Earth's gravitational field? (What else should be considered if using this figure in any scientific argument?)
4. (1 pt) What is the force of the Sun on the Earth and the Earth on the Sun? Is the gravitational forces the same? Are the two force interactions opposite in direction? Why?
5. (5 pt) At what distance from the Earth does the gravitational field of the Sun cancel out that of the Earth? Use both methods below to calculate and compare the results. Only carry two significant figures in the calculation. What would happen if one placed a satellite at this point of zero gravity in a synchronise orbit with the Earth's orbit around the Sun?
6. (2 pt) Why are the space-shuttle astronauts weightless? Hint: Centripetal acceleration and orbital trajectory.
7. (2 pt) Why do two objects with different inertial mass fall at the same rate in a vacuum where gravity is only force acting on the objects and the gravity is constant. Hint: use definition of of inertial force and gravitational force.

Answers

1.

Planet	Mercury	Venus	Earth	Mars	Jupiter
k	2.2×10^{13}	3.3×10^{14}	4.0×10^{14}	4.3×10^{13}	1.3×10^{17}
Mass (kg)	3.3×10^{23}	4.9×10^{24}	5.98×10^{24}	6.4×10^{23}	1.9×10^{27}

$$2. \quad g = \frac{1.33 \times 10^{20}}{(1.5 \times 10^{11})^2} = 5.91 \times 10^{-3} \text{ ms}^{-2}$$

$$3. \quad g = \frac{4.0 \times 10^{14}}{(1.5 \times 10^{11})^2} = 1.8 \times 10^{-8} \text{ ms}^{-2}$$

$$4. \quad \text{Force of sun on earth: } F = m_e * g_{\text{sun}} = (5.98 \times 10^{24} \text{ kg}) * (5.91 \times 10^{-3} \text{ m/s}^2) = 3.55 \times 10^{22} \text{ N.}$$

$$\text{Force of earth on sun: } F = m_s * g_{\text{earth}} = (1.99 \times 10^{30} \text{ kg}) * (1.8 \times 10^{-8} \text{ m/s}^2) = 3.58 \times 10^{22} \text{ N.}$$

$$5. \quad 2.59 \times 10^8 = 259,000 \text{ km from Earth on line connecting centers of sun-earth.}$$

Hints for Questions 5

Method 1. Consider a point in space somewhere between the two bodies where the gravitational fields of each body are equal in strength and thus cancel each other out.

At this point the equation for the gravitational fields of each body are equal. For the Earth-Sun system with r_e being the distance from the earth's center to the neutral gravity point and r_s being the distance from the sun's center to the neutral gravity point, the equation is:

$$\frac{Gm_E}{r_E^2} = \frac{Gm_s}{r_s^2} \Rightarrow \frac{m_E}{r_E^2} = \frac{m_s}{r_s^2} \Rightarrow \frac{r_E}{r_s} = \sqrt{\frac{m_E}{m_s}}$$

By rearranging to give masses on one side of the equation, and radial distances on the other, we find an expression for the ratio of the masses of the bodies. If we know these masses we can calculate the ratios value.

The square root of this ratio, is equal to the ratio of the distances between the bodies. Since we know the total distance between the bodies, we can find a value for the distance of our point in space from either body.

Method 2. Define: x = distance from Earth to neutral gravity point, y = distance between sun and earth. What is $(y-x)$ equal to? Set gravitational acceleration equal at distance x from the earth and distance $(y-x)$ from the sun.

$$\frac{k_E}{x^2} = \frac{k_s}{(y-x)^2}$$

$$\Rightarrow k_E (y^2 - 2xy + x^2) = k_s x^2$$

$$\Rightarrow k_E y^2 - 2k_E xy + (k_E - k_s)x^2 = 0$$

$$a = (k_E - k_s) \quad b = -2k_E y \quad c = k_E y^2$$

Quadratic formula:

$$x = \frac{2k_E y \pm \sqrt{(-2k_E y)^2 - 4 * (k_E - k_s) * k_E y^2}}{2 * (k_E - k_s)}$$